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TeLEx: learning signal temporal logic from positive examples using tightness metric

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Abstract
We propose a novel passive learning approach, TeLex, to infer signal temporal logic (STL) formulas that characterize the behavior of a dynamical system using only observed signal traces of the system. First, we present a template-driven learning approach that requires two inputs: a set of observed traces and a template STL formula. The unknown parameters in the template can include time-bounds of the temporal operators, as well as the thresholds in the inequality predicates. TeLEx finds the value of the unknown parameters such that the synthesized STL property is satisfied by all the provided traces and it is *tight*. This requirement of *tightness* is essential to generating interesting properties when only positive examples are provided and there is no option to actively query the dynamical system to discover the boundaries of legal behavior. We propose a novel quantitative semantics for satisfaction of STL properties which enables TeLEx to learn tight STL properties without multidimensional optimization. The proposed new metric is also smooth. This is critical to enable the use of gradient-based numerical optimization engines and it produces a 30x to 100x speed-up with respect to the state-of-art gradient-free optimization. Second, we present a novel technique for automatically learning the structure of the STL formula by incrementally constructing more complex formula guided by the robustness metric of subformula. We demonstrate the effectiveness of the overall approach for learning STL formulas from only positive examples on a set of synthetic and real-world benchmarks.

Keywords Signal temporal logic · Specification mining · Transparent machine learning · Interpretable machine learning · Cyber-physical system · Autonomous system

1 Introduction

Signal Temporal Logic (STL) [30] is a discrete linear time temporal logic used to reason about the future evolution of a continuous time behavior. Generally, this formalism is useful in describing the behaviors of trajectories of differential equations or hybrid models. Several approaches [16,22,23,29,37,38] have been recently proposed to automatically design sys-

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tems and controllers to satisfy given temporal logic specifications. But practical systems are still often created as an assembly of components—some of which are manually designed. Further, many practical systems also include the physical plant, and the overall property of such systems are not known a-priori. Consequently, specification mining has emerged as an effective approach to create abstractions of monitored behavior to better understand complex systems, particularly in autonomy and robotics. STL is a natural choice for mining these specifications. In this paper, we use bounded-time variant of STL where all temporal operators are associated with lower and upper time-bounds. The truth of formulas in bounded-time STL can be assessed using only finite length trajectories. Our goal is to mine specifications by observing a system for a finite time and hence, bounded-time STL is a suitable target.

Existing approaches to learning STL properties fall into two categories. The approaches in the first category are classifier-learning techniques which rely on the presence of both positive and negative examples to learn STL formula as a classifier. The approaches in the second category are active-learning approaches that require the capability to experiment with the system to actively try falsifying candidate STL properties in order to obtain counterexamples. In this paper, we address the problem of learning STL properties where negative examples are not provided and it is not possible to actively experiment with the system in a safe manner. For example, learning properties of a vehicle-deployed autonomous driving system must rely on only positive examples. We neither have easy access to negative example trajectories that the system will never execute nor have an easy way to design safe experiments for falsifying properties. The boundary of legal behavior needs to be inferred using only positive examples.

We propose a novel technique, `TeLEx` that addresses this challenge of data-driven learning of STL formulas from just positive example trajectories. An initial learning bias is provided to `TeLEx` as a template formula. `TeLEx` is restricted to learning parameters of the provided template STL formula and not its structure. `TeLEx` does not have access to either negative examples or the model of the system for falsification. Thus, the boundaries of legal behavior are not directly available. It has to be inferred just from positive examples. The challenge is to avoid over-generalization in absence of negative examples or counterexamples obtained from active falsification. `TeLEx` addresses this research gap of mining temporal specifications of systems where active experimentation is not possible and failing traces (negative examples) are not available.

Learning from just positive examples has been studied in machine learning [9,24,28,32,34,42] in different contexts such as learning geometric shapes, logic programs and recursive languages. These approaches often employ some metric that characterizes the complexity/simplicity of the concepts to avoid over-generalization. They use this metric to find the simplest concept consistent with the positive examples. For example, an approach for learning convex polytopes from positive examples, would find the tightest convex hull of the given examples. To the best of our knowledge, `TeLEx` is the first approach to learn signal temporal logic properties from just positive examples. We use a novel quantitative metric to define how tightly an STL formula is satisfied by a set of positive examples. Based on this metric, `TeLEx` infers the tightest STL property consistent with the provided example traces.

Quantitative satisfiability metrics for STL properties have been previously proposed in literature. These metric capture robustness or average-robustness of satisfiability of an STL formulae over trajectories. These metrics do not capture the tightness of satisfiability. `TeLEx` uses a novel quantitative metric that measures the tightness of satisfiability of STL formulas over the traces. This metric uses smooth functions to represent predicates and temporal oper-

ators. This keeps the metric differentiable, which would not be possible by just taking the absolute value of standard robustness-metric or directly using the qualitative metric. While sigmoid and exponential-like functions are often used in fields such as deep-learning which rely on numerical-optimization, TeLEx is the first to use these to *smoothly* represent tight-satisfiability of STL formulas. The smoothness of the proposed metric allows the effective use of gradient-based numerical optimization techniques. This improves scalability compared to gradient-free optimization techniques traditionally used for STL learning. TeLEx can be used with a number of different numerical optimization back-ends to synthesize parameters that minimize the new metric over positive examples, and thus, learn a tight STL formula consistent with all the traces.

This paper is an extended version of our previous work [25]. The technical extensions include automatically learning structure of the STL formula and an extended experimental evaluation that compares the proposed approach against state of the art techniques that use genetic algorithms for learning formula structure. The overall technical contributions of the paper are as follows:

- We propose a new quantitative *tightness* metric for measuring how tightly an STL formula is satisfied by a trajectory. This metric is negative when the formula is not satisfied and positive when the formula is satisfied. The metric peaks when the STL formula is just satisfied by the trajectory.
- We present a technique to efficiently learn parametric STL from positive examples and an STL template, using gradient descent methods by exploiting the smoothness of proposed tightness metric.
- We propose a novel approach to automatically learn structure of the STL by incrementally constructing more complex STL formula guided by the robustness metric of satisfiability until a tight fitting STL formula is learned.
- We evaluate the proposed STL on a set of synthetic and real-world benchmarks.

The rest of the paper is organized as follows. We briefly summarize the background on STL and its quantitative semantics given by a robustness metric in Sect. 2. We discuss relevant related work on learning temporal logic properties, quantitative metrics for satisfiability of STL and machine learning literature on learning from positive examples in Sect. 3, and contrast them with the approach proposed in this paper. We present the new tightness metric to learn STL formulas from positive examples in Sect. 4 along with the overall gradient-based approach for learning parameterized STL formulas. We also present a novel approach to automatically learn structure of the STL formulas in Sect. 5. We discuss the experimental results in Sect. 6 and conclude in Sect. 7.

2 Preliminaries

We present some preliminary concepts and definitions used in our work. We begin by reviewing interval arithmetic which will be used in defining the semantics of STL.

Definition 1 An interval I is a convex subset of $\mathbb{R}$. A singular interval $[a, a]$ contains exactly one point and $\emptyset$ denotes empty interval. Let $I = [a, b]$, $I_1 = [a_1, b_1]$, and $I_2 = [a_2, b_2]$ be three closed intervals. Then,

1. $-I = [-b, -a]$
2. $c + I = [c + a, c + b]$
3. $I_1 \oplus I_2 = [a_1 + a_2, b_1 + b_2]$

4. $\min(I_1, I_2) = [\min(a_1, a_2), \min(b_1, b_2)]$
5. $I_1 \cap I_2 = [\max(a_1, a_2), \min(b_1, b_2)]$ if $\max(a_1, a_2) \leq \min(b_1, b_2)$ and $\emptyset$ o.w.

These definitions for various operations are naturally extended to closed, open-closed, and closed-open intervals.

Definition 2 A time domain ST is a finite or infinite set of time instants such that $ST \subseteq \mathbb{R}^{\geq 0}$ with $0 \in ST$. A signal or signal-trace τ is a function from ST to a domain $\mathcal{X} \subseteq \mathbb{R}$. We assume the domain of all signals to be $\mathbb{R}$ to simplify notation. We also refer to signal-trace as simply trace or trajectory.

Monitors used in cyber-physical systems, as well as simulation frameworks, typically provide signal values at discrete time instants due to discrete sampling, or due to limitations of numerical integration techniques. The actual signal can be reconstructed from discrete-time samples using some form of interpolation. In this paper, we assume constant interpolation to reconstruct the signal $\tau(t)$, that is, given a sequence of time-value pairs $(t_0, x_0), \ldots, (t_n, x_n)$, for all $t \in [t_0, t_n)$, we define $\tau(t) = x_i$ if $t \in [t_i, t_{i+1})$, and $\tau(t_n) = x_n$.

The signal temporal logic (STL) formula are used to describe properties of signals. The syntax of STL is given as follows:

Definition 3 A formula $\phi \in \mathcal{F}$ of bounded-time STL is defined as follows:

$$\phi := \bot \mid \top \mid \mu \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \phi \mathbf{U}_{[t_1,t_2]}\phi \mid \mathbf{F}_{[t_1,t_2]}\phi \mid \mathbf{G}_{[t_1,t_2]}\phi$$

where $0 \leq t_1 < t_2 < \infty$ and the atomic predicates $\mu : \mathbb{R}^n \to \{\top, \bot\}$ are inequalities on a set X of n signals, that is, $\mu(X)$ is of the form $g(X) \geq \alpha$, where $\alpha \in \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}$ is a continuous function.

The eventually $\mathbf{F}$ and globally $\mathbf{G}$ operators are shorthands for $\top \mathbf{U}_{[t_1,t_2]}\phi$ and $\neg(\top \mathbf{U}_{[t_1,t_2]}\neg\phi)$ respectively. We keep them, nonetheless, to aid clarity when presenting the different ways of assigning semantics to these operators. We refer to [12,30], and the survey in [31], for detailed discussion on STL.

We briefly summarize its qualitative semantics in Definition 4.

Let $\mathcal{T}$ denote the set of all signal-traces.

Definition 4 The qualitative semantics of STL formulas is given by the function $\psi : \mathcal{F} \times \mathcal{T} \times ST \to \texttt{Bool}$ that maps an STL formula ϕ, a given signal-trace $\tau \in \mathcal{T}$, and a time $t \in ST$ to a Boolean value (True $\top$, or False $\bot$) such that

- $\psi(\top, \tau, t) = \top$
- $\psi(\mu, \tau, t) = \mu(\tau(t))$
- $\psi(\neg\phi, \tau, t) = \neg\psi(\phi, \tau, t)$
- $\psi(\phi_1 \vee \phi_2, \tau, t) = \psi(\phi_1, \tau, t) \vee \psi(\phi_2, \tau, t))$
- $\psi(\phi_1 \wedge \phi_2, \tau, t) = \psi(\phi_1, \tau, t) \wedge \psi(\phi_2, \tau, t))$
- $\psi(\mathbf{F}_{[t_1,t_2]}\phi, \tau, t) = \exists t' \in [t + t_1, t + t_2]\ \psi(\phi, \tau, t')$
- $\psi(\mathbf{G}_{[t_1,t_2]}\phi, \tau, t) = \forall t' \in [t + t_1, t + t_2]\ \psi(\phi, \tau, t')$
- $\psi(\phi_1 \mathbf{U}_{[t_1,t_2]}\phi_2, \tau, t) = \exists t' \in [t + t_1, t + t_2]\ (\psi(\phi_2, \tau, t') \ \wedge\ \forall t'' \in [t, t')\ \psi(\phi_1, \tau, t''))$

Motivated by the need to define how robustly a trace satisfies a formula, formulas in STL were given a quantitative semantics, where formulas are interpreted over numbers such that positive numbers indicate that the formula is True, and negative numbers indicate falsehood. We summarize the quantitative semantics (robustness metric) from [13,15] below.

Definition 5 The robustness metric ρ maps an STL formula $\phi \in \mathcal{F}$, a signal trace $\tau \in \mathcal{T}$, and a time $t \in ST$ to a real value, that is, $\rho : \mathcal{F} \times \mathcal{T} \times ST \rightarrow \mathbb{R} \cup \{\infty, -\infty\}$ such that:

- $\rho(\top, \tau, t) = +\infty$
- $\rho(\mu, \tau, t) = g(\tau(t)) - \alpha$ where $\mu(X)$ is $g(X) \geq \alpha$
- $\rho(\neg\phi, \tau, t) = -\rho(\phi, \tau, t)$
- $\rho(\phi_1 \vee \phi_2, \tau, t) = \max(\rho(\phi_1, \tau, t), \rho(\phi_2, \tau, t))$
- $\rho(\phi_1 \wedge \phi_2, \tau, t) = \min(\rho(\phi_1, \tau, t), \rho(\phi_2, \tau, t))$
- $\rho(\mathbf{F}_{[t_1,t_2]}\phi, \tau, t) = \sup_{t' \in [t+t_1, t+t_2]} \rho(\phi, \tau, t')$
- $\rho(\mathbf{G}_{[t_1,t_2]}\phi, \tau, t) = \inf_{t' \in [t+t_1, t+t_2]} \rho(\phi, \tau, t')$
- $\rho(\phi_1 \mathbf{U}_{[t_1,t_2]}\phi_2, \tau, t) = \sup_{t' \in [t+t_1, t+t_2]} (\min(\rho(\phi_2, \tau, t'), \inf_{t'' \in [t, t')} \rho(\phi_1, \tau, t'')))$

A STL formula ϕ is satisfied by a trace τ at time t, that is, $\psi(\phi, \tau, t) = \top$ if and only if $\rho(\phi, \tau, t) \geq 0$. Intuitively,

ρ quantifies the *degree of satisfiability*. A large positive value indicates that the formula ϕ is robustly satisfied by the trace τ at time t, a positive value close to zero suggests that $\tau(t)$ satisfies ϕ but it is close to violating ϕ, and a negative value indicates that the formula ϕ is violated by $\tau(t)$. This has motivated its use in learning STL formulae for specification mining [7,13,20,26], diagnosis [27], falsification [1,2,6], and system synthesis [4,11,38].

We also define a complexity measure χ for STL formula which is used in searching for incrementally more complex STL formulas which discovering the formula structure. Intuitively, the complexity measure captures the level of temporal nesting in an STL formula.

Definition 6 The complexity measure χ maps an STL formula $\phi \in \mathcal{F}$ to a non-negative integer value, that is, $\chi : \mathcal{F} \rightarrow \mathbb{Z}^+$ such that:

- $\chi(\top) = 0$, $\chi(\mu) = 0$ where $\mu(X)$ is $g(X) \geq \alpha$
- $\chi(\neg\phi) = 1 + \chi(\phi)$
- $\chi(\phi_1 \vee \phi_2) = \chi(\phi_1) + \chi(\phi_2)$, $\chi(\phi_1 \wedge \phi_2) = \chi(\phi_1) + \chi(\phi_2)$
- $\chi(\mathbf{F}_{[t_1,t_2]}\phi) = 1 + \chi(\phi)$, $\chi(\mathbf{G}_{[t_1,t_2]}\phi) = 1 + \chi(\phi)$
- $\chi(\phi_1 \mathbf{U}_{[t_1,t_2]}\phi_2) = \chi(\phi_2) + \chi(\phi_1)$

Finally, we define the duration of a bounded-time STL formula which intuitively corresponds to the period of time required to evaluate the truth of the formula over a given trajectory.

Definition 7 The duration Δ maps an STL formula $\phi \in \mathcal{F}$ to a non-negative real value, that is, $\Delta : \mathcal{F} \rightarrow \mathbb{R}$ such that:

- $\Delta(\top) = 0$, $\Delta(\mu) = 0 \text{where} \mu(X) \text{is} g(X) \geq \alpha \Delta(\neg\phi) = 0$
- $\Delta(\phi_1 \vee \phi_2) = \max(\Delta(\phi_1), \Delta(\phi_2))$
- $\Delta(\phi_1 \wedge \phi_2) = \max(\Delta(\phi_1), \Delta(\phi_2))$
- $\Delta(\mathbf{F}_{[t_1,t_2]}\phi) = (t_2 - t_1) + \Delta(\phi)$
- $\Delta(\mathbf{G}_{[t_1,t_2]}\phi) = (t_2 - t_1) + \Delta(\phi)$
- $\Delta(\phi_1 \mathbf{U}_{[t_1,t_2]}\phi_2) = (t_2 - t_1) + \max(\Delta(\phi_2) + \Delta(\phi_1))$

3 Related work

In this section, we summarize related work on learning STL formulas and contrast them to the approach presented in this paper. We categorize related work into three groups: learning

STL formula, quantitative metrics for temporal logic and learning concepts from positive examples.

3.1 Learning STL formula

Existing techniques for learning STL formulas can be broadly classified into active and passive methods. Active STL learning methods rely on availability of a simulation model on which candidate temporal properties can be falsified [1,3,6,41]. This generates counterexamples. Since these models are often complex executable models, black-box optimization techniques such as simulated annealing are used in falsification of candidate temporal logic properties. If the falsification succeeds, the incorrect parameter values are eliminated and the obtained negative example is used in the next iteration of inferring new candidate parameters values of the temporal logic property. We address a different problem of learning signal temporal logic formula when the simulation model is not available. Further, instead of using gradient-free optimization methods such as simulated annealing, Monte Carlo and ant colony optimization to falsify models, we use more scalable gradient-based numerical optimization methods to infer tightest STL property consistent with a given set of traces. Gradient-based methods for falsification [2] have also been proposed recently to exploit the differentiable nature of simulation models but our approach does not have access to a simulation model. Instead, we define a *smooth* tightness metric for satisfiability of STL properties, and use gradient-based methods to search over the parameter space of STL formulae. [35] presents a smoothness metric that uses Meyer wavelet expansion. In contrast, we use custom smoothening functions in defining the tightness metric.

Passive data driven approaches for learning parametric STL formula from positive and negative example traces have also been proposed in literature. Learning STL formula is reduced to a two class supervised classification problem [7,17,27] that is solved using a mixture of discrete and continuous optimization using decisions trees and simulated annealing. A model based approach that relies on statistical induction of models before learning STL formulae is presented in [7]. In contrast, TeLEx addresses the problem of passive learning of STL formulae in presence of only positive examples.

Learning the structure of STL formula [7,8,17,27] along with parameters using a dataset comprising of positive and negative examples, has also been studied in literature. Typically, the problem is addressed in two steps, learning the structure followed by the synthesis of parameters. The structure of the STL formula in [27] is learned by exploring a directed acyclic graph. In [17], a decision tree approach is used to learn both the structure and the parameters. Genetic algorithms have also been used to synthesize the STL formula structure [7,8]. In contrast to these approaches, this paper focuses on learning STL formula using a dataset of only positive examples and hence, decision-tree based approaches or those using fitness functions for genetic algorithm corresponding to the accuracy of separating positive and negative examples, are not applicable. Our approach to learn the structure of STL formula is the first to exploit robustness metric to incrementally construct more complex STL formula.

3.2 Metrics for STL satisfiability

Signal temporal logic was introduced [13,30] within the context of monitoring temporal properties of signals. It is possible to quantify the degree of satisfiability of an STL property on a signal trace, thus going beyond the Boolean interpretation. Robustness metric was proposed [13,15] to provide such a quantitative metric, as described in Sect. 2. Intuitively,

this metric captures the closest distance between the signal trace and the boundary of set of signals satisfying the STL property. This is the worst-case measure of degree of satisfiability. More recently, an *average robustness metric* has also been proposed [29] in the context of task and motion planning application where the `min` (`inf`) operator in the metric definition for globally properties is replaced by an averaging operator. This allows more efficient encoding to linear programs for certain planning problems. These metrics are monotonic, that is, the measure is higher for formulas that are more robustly satisfiable.

If we use robustness metric to learn STL properties from a set of positive example traces, then we would learn very weak properties.

This is because a weaker STL property would have a higher robustness value for any given set of positive example signal traces. For example, even if $G(x > 0)$ holds for a given set of traces, the formula $G(x > -100)$ holds more robustly, and would be preferred if we optimized for the standard robustness metric.

Hence, in this paper, we define a new metric that captures *tight satisfiability* of an STL property over positive example traces.

A possible approach for finding a tight formula would be to seek a formula that minimizes the absolute-value of the robustness-metric. However, this is not ideal because the absolute-value function is non-differentiable at the optimum and hence, optimizing such a metric would be very challenging. Our proposed novel metric uses smooth functions, such as sigmoid and exponentials, to model tight-satisfiability while still retaining differentiability to aid optimization.

3.3 Learning from positive examples

Learning from positive examples has been investigated extensively in machine learning. Gold et al [18] showed that even learning regular languages from a class with at least one infinite language is not possible with only positive examples in a deterministic setting. Horining [19] considered the case of stochastic context-free grammars and assumed that the positive examples were generated by sampling from the unknown grammar according to the probabilities assigned to the productions. He proved that such positive examples could be used to converge to the correct grammar in the limit with probability one. Angluin [5] generalized these results to identifying any unknown formal language in the limit with probability one as long as positive examples are drawn according to an associated probability distribution. Apart from the literature on language learning,

Muggleton [33] showed that logic programs are learnable with arbitrarily low expected error just from positive examples within a Bayesian framework.

Valiant [40] showed monomials and k-CNF formulas are Probably Approximately Correct (PAC) learnable using only positive examples. While learning from positive examples and its limitations have been studied for other concept classes [24], our approach is the first to consider learning STL properties from positive examples.

4 Learning parameters of STL template

Before we present the proposed approach for learning STL properties from just positive examples, we present a simple motivating example.

4.1 Illustrative example

Let us consider an autonomous vehicle system where the steering angle `ang` and speed `spd` are being observed. Each element of the observed trace is a tuple of the form (`timestamp`, `ang`, `spd`). We would like to learn an STL property with the template:

$$\phi = |\texttt{ang}| \geq 0.2 \Rightarrow F_{[0,6]}\texttt{spd} \leq \alpha$$

which intuitively means that we would like to learn the minimum speed α reached within 6 s of initiating a turn. Let us consider a timestamped signal trace:

$$\tau = (0, 0.1, 15), (2, 0.2, 14), (4, 0.3, 12), (6, 0.35, 10), (8, 0.4, 8), \ldots$$

For this trace, we notice that

$$(|\texttt{ang}| \geq 0.2 \Rightarrow F_{[0,6]}\texttt{spd} \leq 8)$$

would tightly fit the data. But if we used the robustness metric for optimization, increasing the value of α would be preferred since it increases the robustness value. The robustness metric value for the instantiated template ϕ and the trajectory τ is $\rho(\phi, \tau, 0) = 0$ when $\alpha = 8$, $\rho(\phi, \tau, 0) = 2$ when $\alpha = 10$, $\rho(\phi, \tau, 0) = 992$ when $\alpha = 1000$, and so on. A weaker property such as

$$|\texttt{ang}| \geq 0.2 \Rightarrow F_{[0,6]}\texttt{spd} \leq 1000$$

has higher robustness score than the tight property

$$|\texttt{ang}| \geq 0.2 \Rightarrow F_{[0,6]}\texttt{spd} \leq 8$$

but clearly, the latter is a more fitting description of the observed behavior.

4.2 Problem definition

We next present some definitions essential to formulating the problem of learning STL properties from positive examples.

Definition 8 A template STL formula $\phi(p_1, p_2, \ldots, p_k)$ with k unknown parameters is a negation-free bounded-time signal temporal logic formula with the syntax in Definition 3 where some of the time bounds of temporal operators and thresholds of atomic predicates are not constants but instead, free parameters. The parameters are optionally associated with interval constraints providing lower and upper bounds; that is, $l_i \leq p_i \leq u_i$ for $1 \leq i \leq k$ where l_i, u_i are constant bounds.

Note that we assume templates are negation free. If there are no **U** operator in a formula ϕ, then the negation in $\neg\phi$ can be pushed inside a formula until we are only left with negated atomic predictes. Negated predicates can themselves be rewritten in negation-free form.

We say that an STL formula $\phi(v_1, v_2, \ldots, v_k)$ completes the STL template if the values $v_i \in \mathbb{R}$ for parameters p_i satisfy all the bound constraints on p_i.

Definition 9 Given a temporal logic property $\phi(v_1, v_2, \ldots, v_k)$ that completes a template $\phi(p_1, p_2, \ldots, p_k)$, we define the ϵ-neighborhood of $\phi(v_1, v_2, \ldots, v_k)$ as $\mathcal{N}_\epsilon(\phi(v_1, v_2, \ldots, v_k)) = \{\phi(v'_1, v'_2, \ldots, v'_k)\ s.t.\ |v_i - v'_i| \leq \epsilon\ for\ 1 \leq i \leq k\}$.

We now formally define the problem of learning signal temporal logic formula. The second condition in Definition 10 ensures ϵ-tightness while the first condition ensures that the STL formula is consistent with positive examples.

Definition 10 Given a set of traces $\mathcal{T}$ and template STL $\phi(p_1, p_2, \ldots, p_k)$, the problem of learning ϵ-tight STL formula is to learn the values of the parameters, $p_i = v_i^*$, such that the following is true:

- the STL formula $\phi(v_1^*, v_2^*, \ldots, v_k^*)$ holds over all traces in $\mathcal{T}$, that is,

$$\forall \tau \in \mathcal{T} : \ \tau \models \phi(v_1^*, v_2^*, \ldots, v_k^*)$$

- there exists some $\phi(v_1, v_2, \ldots, v_k) \in \mathcal{N}_\epsilon(\phi(v_1^*, v_2^*, \ldots, v_k^*))$ that does not hold over at least one trace in $\mathcal{T}$; that is,

$$\exists \tau \in \mathcal{T} : \tau \not\models \phi(v_1, v_2, \ldots, v_k)$$

We have used the notation $\tau \models \phi$ here to denote $\psi(\phi, \tau, 0) = \top$, where ψ is the qualitative semantics presented in Definition 4.

We can solve the problem of learning ϵ-tight STL formulas by formulating the following *constrained multi-objective optimization problem* where minimization is done with respect to the free parameters $p_1, \ldots, p_k$.

$$\begin{gathered}\texttt{minimize } \{|\epsilon_1|, |\epsilon_2|, \ldots, |\epsilon_k|\} \ \ s.t. \\ \epsilon_1 = p_1 - p_1', \epsilon_2 = p_2 - p_2', \ldots, \epsilon_k = p_k - p_k' \\ \forall \tau \in \mathcal{T} \ \tau \models \phi(p_1, p_2, \ldots, p_k), \\ \exists \tau' \in \mathcal{T} \ \tau' \not\models \phi(p_1', p_2', \ldots, p_k')\end{gathered}$$

We can check if the solution of the above problem solves our ϵ-tight learning problem by checking if $\max\{|\epsilon_1|, \ldots, |\epsilon_k|\}$ is less than the desired ϵ (or, we could alternatively change the above optimization problem to a min-max problem).

However, the above optimization problem is very difficult to solve in practice for two reasons:

- First, it requires multi-objective optimization where the number of objectives, k, grows with the number of parameters in the signal temporal logic formula.
- Second, the constraints require checking satisfiability of the bounded-time STL formula over finite traces which is itself an NP hard problem.

The robustness metric for quantitative satisfiability of STL formula allows us to replace satisfiability checking with nonlinear constraints in the above optimization problem.

$$\begin{gathered}\texttt{minimize } \{|\epsilon_1|, |\epsilon_2|, \ldots, |\epsilon_k|\} \ \ s.t. \\ \epsilon_1 = p_1 - p_1', \epsilon_2 = p_2 - p_2', \ldots, \epsilon_k = p_k - p_k' \\ \forall \tau \in \mathcal{T} \ \rho(\phi(p_1, p_2, \ldots, p_k), \tau, 0) \geq 0, \\ \exists \tau' \in \mathcal{T} \ \rho(\phi(p_1', p_2', \ldots, p_k'), \tau', 0) < 0\end{gathered}$$

Next, we notice that the robustness metric is continuous in the parameters p_i corresponding to inequality thresholds and time-bounds and hence, one could expect that we will obtain a reasonable solution for the above problem by solving the following simpler scalar optimization problem:

$$\texttt{minimize}_{p_1, p_2, \ldots, p_k} \ \min_{\tau \in \mathcal{T}} |\rho(\phi(p_1, p_2, \ldots, p_k), \tau, 0)|$$

There are two problems with this approach of solving the tight-STL learning problem using the above optimization problem.

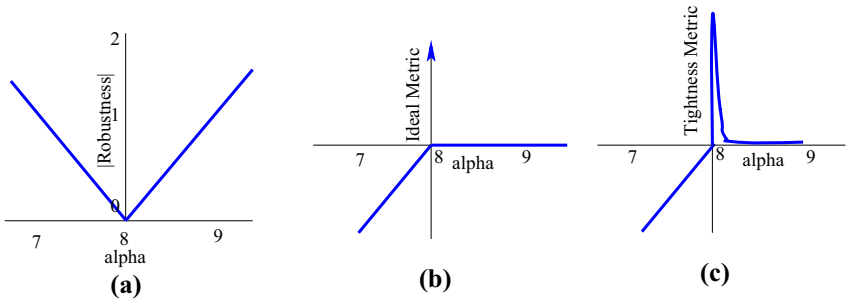


Fig. 1 **a** The absolute value of robustness metric reaches 0 at $\alpha = 8$. It is close to 0 even at 7.99 even though the temporal property corresponding to $\alpha = 7.99$ is violated by the trace. **b** The ideal metric should be negative when $\alpha < 8$ and jump to ∞ when $\alpha = 8$ and drop down to 0 when $\alpha > 8$. **c** A metric which is negative for $\alpha < 8$, reaches its maxima between 8 and $8 + \epsilon$ and then drops to 0

- This optimization problem uses the absolute value of the robustness metric. This metric is generally not differentiable at $\rho(\phi(p_1, p_2, \ldots, p_k)) = 0$.
- Further, if we get an ϵ-approximate solution for the above optimization problem, it no longer guarantees that all traces will satisfy the instantiated template ϕ. This is because the absolute value can be a small positive number even when the actual value is a small negative number.

In Fig. 1, we use the example at the beginning of the section to illustrate the problem. Figure 1b illustrates an ideal metric, because it achieves its maximum at the the boundary of satisfiability and unsatisfiability. Maximizing this metric would yield tight STL property but optimizing such a discontinuous function is difficult. Figure 1c illustrates a more practical incarnation of the ideal metric, which is not discontinuous but still useful to learn ϵ tight STL property. Our main contribution is designing such a metric.

4.3 Tightness metric

We begin by first defining a tightness metric for predicates. We would like the metric to achieve its maximum value at the boundary in order to discover tight STL properties. For a predicate $\mu(\mathbf{x}) := g(\mathbf{x}) \geq \alpha$, recall that the robustness metric is $\rho(\mu, \tau, t) = g(\tau(t)) - \alpha = r$. We would like to define a tightness metric $\theta(\mu, \tau, t)$ such that it is similar to Fig. 1c, and hence we define it to be

$$\frac{1}{r + e^{-\beta r}} - e^{-r}$$

where $\beta \geq 1$ is an adjustable parameter.

This function is plotted in Fig. 2 and it approaches the ideal function in Fig. 1b as β increases albeit at the cost of numerical stability during optimization. This function is smooth (its derivative is defined and also continuous), and hence, is amenable to gradient-based numerical optimization techniques. Finding an ϵ-tight value of α reduces to maximizing θ with appropriate choice of β—lower values of ϵ require higher values of β. Apart from the predicates, the other difficult cases for defining the tightness metric (θ) happen to be the temporal operators. The requirement here is that the metric θ should be defined such that it prefers longer time intervals for globally operator and shorter for eventually operator as illustrated in Fig. 3.

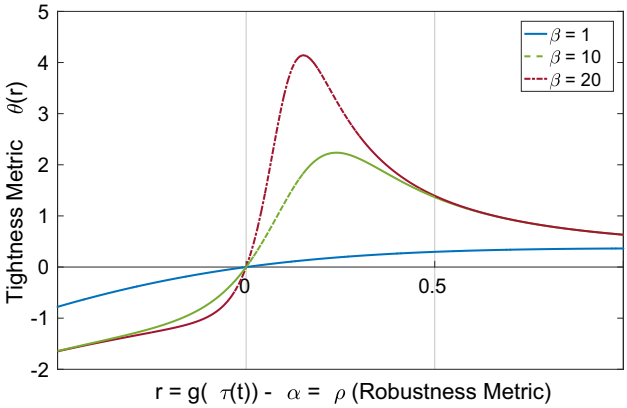


Fig. 2 Tightness metric θ for predicate

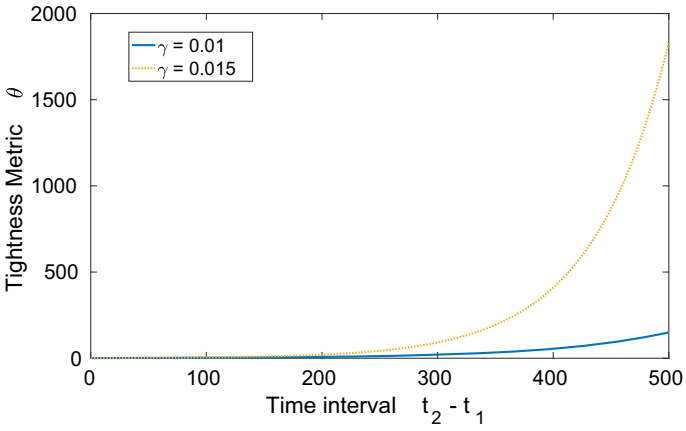
We next formally define the tight quantitative semantics over negation-free STL properties and show how it can be used to formulate the problem of learning consistent and tight STL property as a numerical optimization problem over a single (scalar) cost metric. If the original formula has negation, it is pushed inwards through Boolean combinations, F and G temporal operations, and the inequality in predicate is flipped. Negation can also be pushed inwards through discrete bounded time U operator via case-splitting. Further, since we deal with continuous signals, we consider only non-strict inequalities as predicates and relax strict inequalities if needed.

Definition 11 The tightness metric $\theta : \mathcal{F} \times \mathcal{T} \times ST \mapsto \mathbb{R} \cup \{-\infty, \infty\}$ maps an STL formula $\phi \in \mathcal{F}$, a trace $\tau \in \mathcal{T}$, and a sampled time instance $t \in ST$ to a real value s.t.:

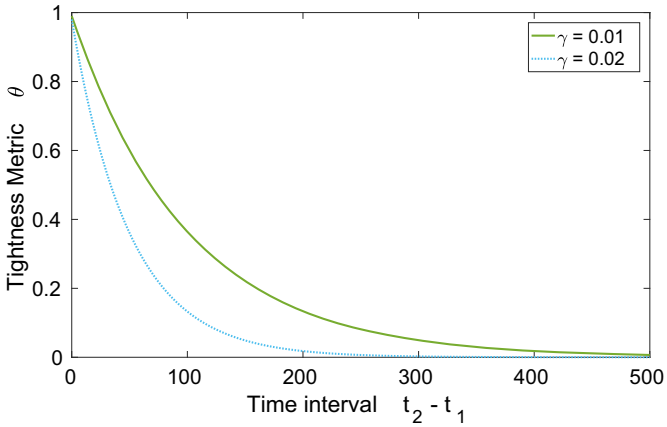
- $\theta(\top, \tau, t) = \infty, \quad \theta(\bot, \tau, t) = -\infty$
- $\theta(\mu, \tau, t) = \mathtt{P}(g(\tau(t)) - \alpha)$ where $\mu(\mathbf{x}) := (g(\mathbf{x}) \geq \alpha)$
- $\theta(\phi_1 \wedge \phi_2, \tau, t) = \min(\theta(\phi_1, \tau, t), \theta(\phi_2, \tau, t))$
- $\theta(\phi_1 \vee \phi_2, \tau, t) = \max(\theta(\phi_1, \tau, t), \theta(\phi_2, \tau, t))$
- $\theta(\mathbf{F}_{[t_1,t_2]}\phi, \tau, t) = \mathtt{C}(\gamma, t_1, t_2) \sup_{t' \in [t+t_1, t+t_2]} \theta(\phi, \tau, t')$
- $\theta(\mathbf{G}_{[t_1,t_2]}\phi, \tau, t) = \mathtt{E}(\gamma, t_1, t_2) \inf_{t' \in [t+t_1, t+t_2)} \theta(\phi, \tau, t')$
- $\theta(\phi_1 \mathbf{U}_{[t_1,t_2]}\phi_2, \tau, t) = \mathtt{E}(\gamma, t_1, t_2) \sup_{t' \in [t+t_1, t+t_2]} (\min(\theta(\phi_2, \tau, t'), \inf_{t'' \in [t, t')} \theta(\phi_1, \tau, t'')))$

where the peak function $\mathtt{P}(r) = \frac{1}{r + e^{-\beta r}} - e^{-r}$, the contraction function $\mathtt{C}(\gamma, t_1, t_2) = \frac{2}{1 + e^{\gamma(t_2 - t_1 + 1)}}$, the expansion function $\mathtt{E}(\gamma, t_1, t_2) = \frac{2}{1 + e^{-\gamma(t_2 - t_1 + 1)}}$, $\beta \geq 1$ is a coefficient chosen to determine sharpness of peak and $\gamma \geq 0$ is a coefficient chosen to trade-off tightness in time vs tightness over predicates for a given time-scale and spread of continuous variables. We choose to use the expansion function $\mathtt{E}$ in the definition of tightness of **U**-formulae. We could replace $\mathtt{E}$ by $\mathtt{C}$ if shorter time-intervals are preferred in the **U**-operator.

If both the time-interval and predicate threshold is unknown for a temporal operator, then there is a choice in either tightening time-intervals and discovering predicates that hold over these or to find tighter predicates over longer (in case of eventually) and shorter (in case of globally) operators. Increasing γ would result in tighter time-intervals. Increasing β would result in tighter predicates.



(a) Globally Operator: $\theta(G_{[t_2-t_1]}\top)$



(b) Eventually Operator: $\theta(F_{[t_2-t_1]}\top)$

Fig. 3 Tightness metric θ

In the following theorem, we summarize the relation between the tightness metric and satisfaction of STL formula.

Theorem 1 *The tightness metric for a given STL formula ϕ, namely $\theta(\phi, \tau, t)$ is nonnegative if and only if τ satisfies ϕ at time t.*

Proof We first show that $\theta(\phi, \tau, t) \geq 0$ if and only if $\rho(\phi, \tau, t) \geq 0$ using structural induction. We have only two nontrivial cases:

- Atomic Predicates: We know that $\frac{1}{r+e^{-\beta r}} - e^{-r} \geq 0$ where $\beta \geq 1$ if and only if $r \geq 0$. Hence, $\theta(\mu, \tau, t) = \frac{1}{r+e^{-\beta r}} - e^{-r} \geq 0$ if and only if $r = g(\tau(t)) - \alpha = \rho(\mu, \tau, t) \geq 0$
- Temporal Operators: $\mathtt{C}(\gamma, t_1, t_2) = \frac{2}{1+e^{\gamma(t_2-t_1+1)}} \geq 0$ for all $t_2 > t_1$ and $\mathtt{E}(\gamma, t_1, t_2) = \frac{2}{1+e^{-\gamma(t_2-t_1+1)}} \geq 0$ for all $t_2 > t_1$. Hence, θ has the same sign as ρ, that is, $\theta(\phi, \tau, t) \geq 0$ if and only if $\rho(\phi, \tau, t) \geq 0$.

Thus, $\theta(\phi, \tau, t) \geq 0$ if and only if $\rho(\phi, \tau, t) \geq 0$ and we know that $\rho(\phi, \tau, t) \geq 0$ if and only if τ satisfies ϕ at time t. □

The theorem above shows that a STL formula ϕ that has positive tightness metric (over all the traces τ in some set $\mathcal{T}$) will also evaluate to `True` in all these traces. But we want a formula that is not only consistent with the traces, but also tight on the traces. The following lemma says that optimizing for the tightness metric results in tight formulas.

Lemma 1 *Given a trace τ and a template STL formula $\phi(p_1, p_2, \ldots, p_k)$ with k unknown parameters (Definition 8), let*

$$(v_1^*, v_2^*, \ldots, v_k^*) = \arg \max_{p_1, p_2, \ldots, p_k} \theta(\phi(p_1, p_2, \ldots, p_k), \tau, 0)$$

be a solution $\mathbf{v}^* = (v_1^*, \ldots, v_k^*)$ such that $\theta(\phi(\mathbf{v}^*), \tau, 0)$ is a finite nonnegative value. Then $\mathbf{v}^*$ is a solution for the ϵ-tight STL learning problem on the singleton set $\{\tau\}$ of traces for any value of ϵ such that $\epsilon > \eta$, where η is no more than the robustness $\rho(\phi(\mathbf{v}^*), \tau, 0)$ of the discovered instantiated formula. The value η can be made arbitrarily small with appropriate choice of β, γ.

Proof We again argue by structural induction over the template ϕ. Since ϕ is negation-free, we have three cases:

- **Case 1** If the top symbol of ϕ is a temporal operator with a time bound $[t_1, t_2]$ such that either t_1 or t_2 is a parameter, then our definition of θ guarantees that the interval $[t_1^*, t_2^*]$ (in the instantiated solution) is maximally elongated or contracted, and hence $\phi(\mathbf{v}^*)$ can be falsified by an ϵ perturbatation to the interval, for any $\epsilon > 0$.
- **Case 2** If ϕ is an atomic predicate, then the robustness measure ρ clearly defines the minimum perturbation required to falsify it.
- **Case 3** If the top symbol of ϕ is $\vee$ or $\wedge$, we can reason inductively one or both of the sub-formulas.

For the second part, note that we can decrease η by choosing a large β and $\gamma > 0$. The value of r at which the function $\frac{1}{r+e^{-\beta r}} - e^{-r}$ peaks monotonically decreases with β and hence, more tight predicates (smaller r) can be learned by increasing β. Hence, η decreases by increasing β. From the definition of `C`, we observe that the function $\frac{2}{1+e^{\gamma(\Delta t+1)}}$ decreases monotonically with γ and the function $\frac{2}{1+e^{-\gamma(\Delta t+1)}}$ increases monotonically with γ. Thus, if $\gamma > 0$, these functions cause us to learn the largest or smallest possible time interval, and hence changing the learned intervals even slightly falsifies the formula. Hence, if $\gamma > 0$, then $\eta = 0$ for formulas that have a parametric temporal operator at the top. □

Theorem 2 *Given a set of traces $\mathcal{T}$ and a template STL formula $\phi(p_1, p_2, \ldots, p_k)$, let*

$$(v_1^*, v_2^*, \ldots, v_k^*) = \arg \max_{p_1, p_2, \ldots, p_k} [\min_{\tau \in \mathcal{T}} \theta(\phi(p_1, p_2, \ldots, p_k), \tau, 0)]$$

define the solution $\mathbf{v}^* = (v_1^*, \ldots, v_k^*)$ such that $\min_{\tau \in \mathcal{T}} \theta(\phi(\mathbf{v}^*), \tau, 0)$ is nonnegative. Then the learnt formula $\phi(\mathbf{v}^*)$ solves the ϵ-tight STL learning problem for a value of ϵ such that $\epsilon > \eta$, where $\eta = \min_{\tau \in \mathcal{T}} \rho(\phi(v_1^*, \ldots, v_k^*), \tau, 0)$ is the standard robustness measure of the discovered instantiated formula. The value η gets no larger by increasing β and γ.

Proof For predicates, η decreases with increasing values of β. From the definition of `C`, we observe that the function $\frac{2}{1+e^{\gamma(\Delta t+1)}}$ decreases monotonically with γ and the function $\frac{2}{1+e^{-\gamma(\Delta t+1)}}$ increases monotonically with γ. Thus, the time-bounds in the temporal operator can be learned more tightly by increasing γ, that is, η reduces with increase in γ. Thus, η decreases with increasing β, γ values for all predicates and temporal operators. □

4.4 Numerical optimization

We use an off-the-shelf solver—quasi-Newton algorithm [14,43] to solve the above optimization problem. It uses gradient during optimization where the search direction in each iteration i is computed as $d_i = -H_i g_i$. H_i is the inverse of the Hessian matrix and g_i is the current derivative. The Hessian is a matrix of second-order partial derivatives of the cost function and describes its local curvature. Due to the smoothness of the defined tightness metric θ, gradient-based optimization techniques are very effective in solving the STL learning problem since both the gradient and the Hessian can be conveniently computed.

We also used the gradient-free optimization to experimentally validate the advantage of smoothness of tightness metric. The optimization engine behind gradient-free optimization is differential evolution [36].

5 Learning structure of STL template

In Sect. 4, STL learning required providing a template. Coming up with this template can be difficult in some applications. We now present an automated approach to learn the structure of the formula. We first define an augmented signal trajectory that extends a given signal trajectory with quantitative robustness metric values for a given set of STL properties.

Definition 12 Given a trajectory τ that maps variables $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$ to real values for a set of time instants, and a set of signal temporal logic properties $\Phi = \{\phi_1, \phi_2, \ldots, \phi_m\}$, an augmented signal τ^{Φ} maps an augmented set of variables $\mathcal{X} = \{x_1, x_2, \ldots, x_n, r_1, r_2, \ldots, r_m\}$ to real values for the time instants in τ such that the value of the newly introduced variable r_j at any time instant is the value of the robustness metric of satisfiability of ϕ_j for τ, that is,

$$r_j(t) = \rho(\phi_j, \tau, t) \text{ for all } t > \Delta(\rho) \text{ and } -\infty \text{ otherwise}$$

We demonstrate an augmented trajectory in Fig. 4. The original trace τ comprises of a single variable x and the only signal temporal logic property used to augment τ is $G_{[0,2]}x \geq 5$. The figure shows the original signal and the new signal corresponding to the robustness value of the STL property. We use augmented trajectories to discover nesting of temporal properties that produce more tightly fitting STL properties for a trace.

We next make an important observation that is central to discover STL formulas with more complex Boolean combination structure. Intuitively, we combine STL sub-formulas with positive robustness values using conjunction and combine STL sub-formulas with negative robustness values using disjunction.

Lemma 2 *Given a trajectory τ and two sub-formulas ϕ_1 and ϕ_2 such that $\rho(\tau, \phi_1, t) \geq 0$ and $\rho(\tau, \phi_2, t) \geq 0$, then $\rho(\tau, \phi_1 \wedge \phi_2, t) \geq 0$*

Proof The proof follows from the definition of robustness metric value. The robustness metric has non-negative value if and only if the sub-formula is satisfied by the trace. If two sub-formula are satisfied by the trace, their conjunction must also be satisfied by it. □

Observation If $\rho(\tau, \phi_1, t) < 0$ and $\rho(\tau, \phi_2, t) < 0$, then it is still possible that $\rho(\tau, \phi_1 \vee \phi_2, t) \geq 0$

Given a set of candidate inequality predicate templates P over the signals and the set of traces $\mathcal{T}$, we now describe an algorithm that incrementally builds more complex STL formula

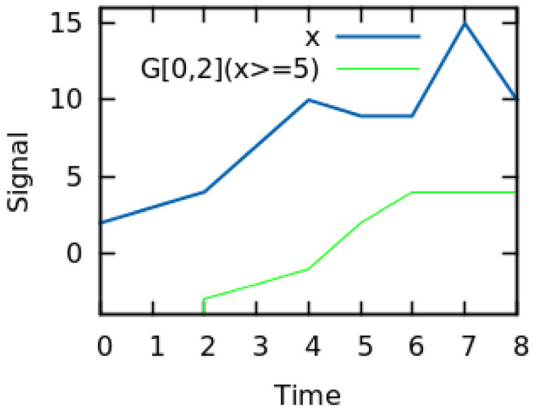


Fig. 4 Augmented trace: x is the variable in the original trace and new signal corresponds to robustness metric value of $G_{[0,2]}x \geq 5$

and learns parameters for them using the parameter learning method described in Sect. 4. The overall procedure is presented in Algorithm 1. It uses the following key sub-procedures:

– $\texttt{Temporal}(P)$: This sub-procedure takes a set of predicates and generates all single-level parametric STL templates, that is,

$$\texttt{Temporal}(P) = \{F_{[a,b]}p \mid p \in P\} \cup \{F_{[a,b]}p \mid p \in P\} \cup \{p_1 U_{[a,b]} p_2 \mid p_1, p_2 \in P\}$$

where a, b denote new parameters in the STL templates.

– $\texttt{Disjunt}(\mathcal{T}, \Phi)$: This sub-procedure takes a set of formula with negative robustness values, and generates disjunct templates, that is,

$$\texttt{Disjunct}(\mathcal{T}, \Phi) = \{\psi_1 \vee \psi_2 \mid \psi_1 \in \texttt{Tmpl}(\phi_1), \psi_2 \in \texttt{Tmpl}(\phi_2) \text{ and } \phi_1, \phi_2 \in \mathcal{R}(\mathcal{T}, \Phi)\}$$

where $\mathcal{R}(\mathcal{T}, \Phi) = \{\phi \mid \rho(\phi, \tau) < 0 \text{ for some } \tau \in \mathcal{T}\}$, and $\texttt{Tmpl}(\phi)$ is a set of STL templates obtained by replacing either the values of temporal operator intervals with parameters or inequality thresholds with parameters. Due to scalability reasons, we do not keep both these parameters as free in a template.

– $\texttt{AugmentPred}(P, \Phi)$: This sub-procedure augments the set of predicates P with new predicates that check whether robustness metric value of the formulas in Φ is greater than zero, that is,

$$\texttt{AugmentPred}(P, \Phi) = P \cup \{r_j \geq 0, r_j < 0 \mid r_j = \rho(\phi_j), \phi_j \in \Phi\}$$

– $\texttt{AugmentTraj}(\mathcal{T}, \Phi)$: This sub-procedure takes a set of trajectories and a set of STL formulas, and generates a new set of augmented trajectories where each original trajectory is augmented with the corresponding robustness metric values of the STL properties, that is,

$$\texttt{Augment}(\mathcal{T}, \Phi) = \{\tau^{\phi} \mid \tau \in \mathcal{T}, \phi \in \Phi\}$$

– $\texttt{Select}(\mathcal{T}, \Phi, i)$: This sub-procedure implements the heuristic to select sub-formula from which more complex formula needs to be constructed at iteration i. If it returns a single formula, the STL learning algorithms terminates reporting this formula as the learned STL. If it becomes empty, then the algorithm terminates reporting that no consistent STL formula can be learned from the given trajectories and with the given set

of predicates. We adopt an heuristic that initially maintains a diverse set and gradually prefers only those formula which are tightly satisfied by traces in $\mathcal{T}$. Specifically,

$$\texttt{Select}(\mathcal{T}, \Phi, i) = \{\phi \mid -mdf(\phi, i) \leq \rho(\phi) \leq mdf(\phi, i)\}$$

where $mdf(\phi, i)$ is monotonically decreasing in both, the complexity measure χ of ϕ and its second argument i. We choose an exponentially decreasing function $mdf(\phi, i) = \rho_{av} e^{-i} / \chi(\phi)$ where ρ_{av} is the average absolute value of the robustness metric for formulas over the trajectories in the previous iteration.

Algorithm 1 Learning STL formulas with structure

```
1: procedure LearnSTL(T, P)
2:    i = 0, Ψ = Temporal(P)
3:    Φ_s = {LearnParameterSTL(T, ψ) | ψ ∈ Ψ}
4:    Φ_0 = Select(T, Φ_a, i)
5:    while |Φ_i| ≥ 1 do
6:       T = AugmentTraj(T, Φ)
7:       P = AugmentPred(P, Φ_i)
8:       Ψ_temporal = Temporal(P)
9:       Ψ_disjunct = Disjunt(T, Φ)
10:      Ψ = Ψ_temporal ∪ Ψ_disjunct
11:      Φ_s = {LearnParameterSTL(T, ψ) | ψ ∈ Ψ}
12:      Φ_i = Select(T, Φ_s, i)
13:      i = i+1
14:   end while
15:   Return Φ_i
16: end procedure
```

The STL formula learned using Algorithm 1 includes auxiliary signals r_j corresponding to the robustness metric values of the sub-formulas. We post process the learned formula to replace predicates $r_j \geq 0$ with the corresponding sub-formula ϕ_j and the predicates $r_j < 0$ with $\neg\phi_j$.

6 Experimental evaluation

The presented approach is implemented in a publicly available tool: TeLEx.[1] We evaluated the effectiveness of `TeLEx` on a number of synthetic and real case-studies. All experiments were conducted on a quad core Intel Core i5-2450M CPU @ 2.50GHz with 3MB cache per core and 4 GB RAM.

We first describe the results of the parameter learning approach presented in Sect. 4 for each of the case-studies, followed by the evaluation of the structure learning algorithm in Sect. 5.

6.1 Temporal bounds on signal $x(t) = t \sin(t^2)$

This case-study was designed to evaluate the scalability of `TeLEx` as well as the tightness of learned STL formulas using a synthetic trajectory for which we already know the

[1] https://github.com/susmitjha/TeLEX.

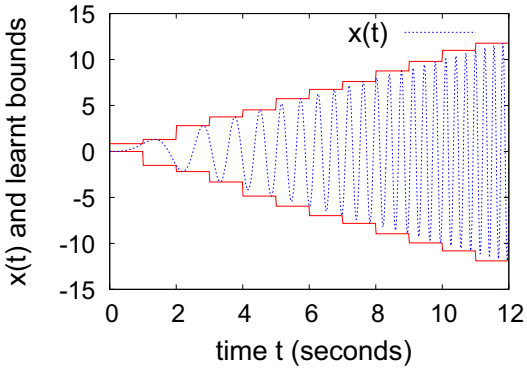


Fig. 5 The signal $x(t)$ and learned parameters of the STL formula

correct answer. We also compare gradient-based TeLEx with gradient-free optimization to demonstrate the utility of smoothness of proposed tightness metric. We consider the signal $x(t) = t \sin(t^2)$.

We consider 12 STL templates of the form:

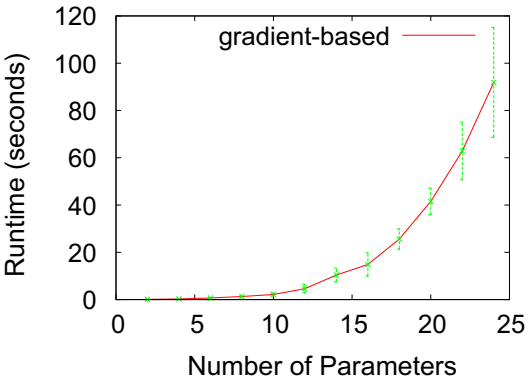
$$template(k) \equiv \bigwedge_{i=0}^{k} (G_{[i,i+1]}(x \leq p_{2i} \wedge x \geq p_{2i+1}))$$

where $k = 0, 1, \ldots, 11$. Thus, the number of parameters in these templates grow from 2 to 24. We repeated learning experiments 10 times in each case since numerical optimization routines are not deterministic.

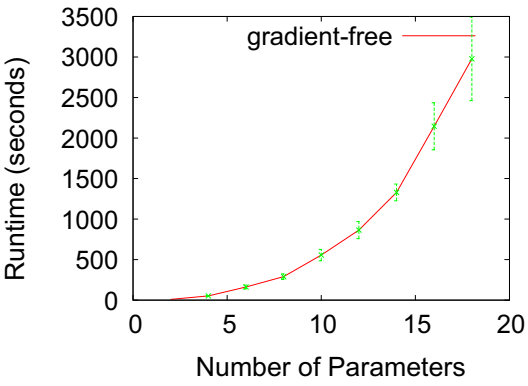
Figure 5 shows the signal trace from time $t = 0$ to $t = 12$ along with the bounds discovered by TeLEx while synthesizing the STL property using template $template(12)$ (the largest template) and gradient-based optimization. The tightness of bounds demonstrates that the learned STL properties are tight (and have very low variance) even with 24 parameters. The robustness values for learned STL properties were always very small (between 0.02 and 0.12). We observed that gradient-free differential evolution also discovered tight properties in all cases (robustness value between (0.06 and 0.35) in which it terminated. Figure 6a, b show the runtime of gradient-based and gradient-free optimization techniques respectively. Gradient-free methods did not terminate in an hour for more than 18 parameters. We plot the mean runtime (along with standard deviation) from 10 runs with respect to the number of parameters being learnt for each of the 12 templates. The variability in runtime (standard deviation plotted as error bars) increases with the number of parameters. We observe a speed-up of 30X-100X using gradient-based approach due to the smoothness of tightness metric (scales of y-axis in Fig. 6a, b are different).

6.2 Two agent surveillance

We consider a two agent surveillance system in which both agents monitor a 10x10 grid as illustrated in Fig. 7. Intruders can pop up at any of the 8 locations marked by circles. But at any point, there are at most two intruders. The two agents are initially at 0, 0 and 10, 10 respectively. The agents follow a simple protocol. At each time-instant, the agents calculate the distance from their current location to the intruders (if any), then they select the intruder closest to them as their target for inspection and move towards it. The target of an agent might



(a) Gradient-based runtime



(b) Gradient-free runtime

Fig. 6 Tightness and scalability of TeLEx using gradient based optimization

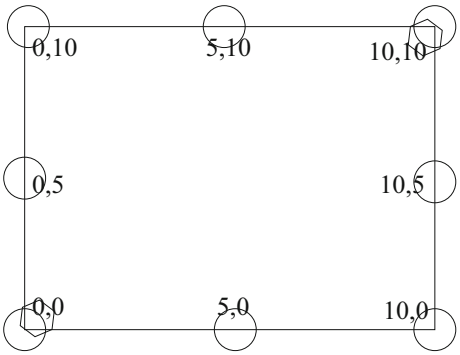
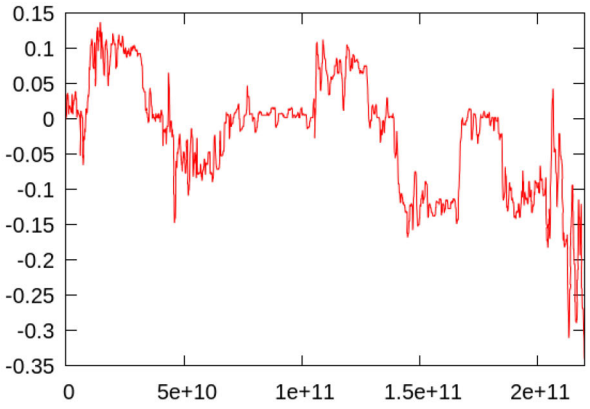
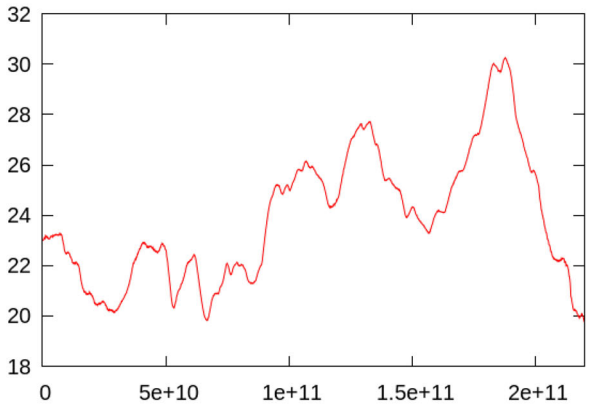


Fig. 7 Two agent surveillance

change while moving (when second intruder pops up and it is closer to the agent moving towards first). After an intruder location is inspected, it is considered neutralized and the agent stays there until new target emerges. The simulator for this simple surveillance protocol is



(a) Angle



(b) Speed

Fig. 8 Angle and speed for a subset of Udacity data

available at the tool website.[2] We simulated this for 1000 time-steps and then used `TeLEx` to learn STL corresponding to the following two properties.

- The maximum time between intruder popping up and being neutralized is 39.001 time-steps.
- The distance between the two agents is at least 4.998. This non-collision between agents is an emergent property due to "move-to-closest" policy of agents and the fact that there are at most two intruders at any given time.

6.3 Udacity autonomous-car driving public data-set

In this case-study, we use the data made available publicly by Udacity as a part of its second challenge for autonomous driving.[3] The data corresponds to an instrumented car (2016 Lin-

[2] https://github.com/susmitjha/TeLEX/blob/master/tests/twoagent.py.

[3] https://github.com/udacity/self-driving-car/tree/master/challenges/challenge-2.

coln MKZ) driving along El Camino Real (a major road in San Francisco Bay Area) starting from the Udacity office in Mountain View and moving north towards San Francisco. We use HMB_1 data-set which is a 221 s snippet with a total of over 13205 samples. It has a mixture of turns and straight driving.

The data-set includes steering angle, applied torque, speed, throttle, brake, GPS and image. For our purpose, we focus on non-image data. The goal of this data-set is to provide real-world training sample for autonomous driving. Figure 8 shows how the angle and speed vary in the Udacity data-set.

We use the tight STL learning approach presented in this paper to learn temporal properties relating angle, torque and speed. Such learned temporal properties could have several utilities. It could be used to examine whether a driving pattern (autonomous or manual) is too conservative or too risky. It could be used to extract sensible logical relations that must hold between different control inputs (say, speed and angle) from good manual driving data, and then enforce these temporal properties on autonomous driving systems. It could also be used to compare different autonomous driving solutions. We are interested in the following set of properties and we present the result of extracting these using `TeLEx` . We would like the robustness metric to be as close to 0 as possible and in all experiments below, we found it to be below 0.005.

1. The speed of the car must be below some upper bound $a \in [15, 25]$ if the angle is larger than 0.2 or below -0.2. Intuitively, this property captures required slowing down of the car when making a significant turn.

Template STL:	$G[0, 2.2e11](((angle \geq 0.2)\|(angle \leq -0.2)) \Rightarrow (speed \leq a?15; 25))$
Synthesized STL:	$G[0.0, 2.2e11](((angle \geq 0.2)\|(angle \leq -0.2)) \Rightarrow (speed \leq 22.01))$
Performance:	Tightness Metric $= 0.067$ Robustness Metric $= 0.004$
Runtime:	8.64 s

2. Similar to the property above, the speed of the car must be low while applying a large torque (say, more than 1.6). Usually, torque is applied to turn along with brake when driving safely to avoid slipping.

Template STL:	$G[0, 2.2e11](((torque \geq 1.6)\|(torque \leq -1.6)) \Rightarrow (speed \leq a?15; 25))$
Synthesized STL:	$G[0.0, 2.2e11](((torque \geq 1.6)\|(torque \leq -1.6)) \Rightarrow (speed \leq 23.64))$
Performance:	Tightness Metric $= 0.221$ Robustness Metric $= 0.005$
Runtime:	10.12 s

3. Another property of interest is to ensure that when the turn angle is high (say, above 0.06), the magnitude of negative torque applied is below a threshold. This avoids unsafe driving behavior of making late sharp compensation torques to avoid wide turns.

Template STL:	$G[0, 2.2e11]((angle \geq 0.06) \Rightarrow (torque \geq b? - 2; -0.5))$
Synthesized STL:	$G[0.0, 2.2e11]((angle \geq 0.06) \Rightarrow (torque \geq -1.06))$
Performance:	Tightness Metric $= 0.113$ Robustness Metric $= 0.003$
Runtime:	7.30 s

4. Similarly, when the turn angle is low (say, below -0.06), the magnitude of positive torque applied is below a threshold to avoid late sharp compensating torques.

Template STL:	$G[0, 2.2e11]((angle \leq -0.06) \Rightarrow (torque \leq b?0.5; 2))$
Synthesized STL:	$G[0.0, 2.2e11]((angle \leq -0.06) \Rightarrow (torque \leq 1.25))$
Performance:	Tightness Metric = 0.472
	Robustness Metric = 0.002
Runtime:	5.00 s

5. The torque also must not be so low that the turns are very slow and so, we require that application of negative torque should decrease the angle below a threshold within some fixed time.

Template STL:	$G[0, 2.2e11]((torque \leq 0.0) \Rightarrow F[0.0, 1.2e8](angle \leq a? - 1; 1))$
Synthesized STL:	$G[0.0, 2.2e11]((torque \leq 0.0) \Rightarrow F[0.0, 1.2e8](angle \leq 0.01))$
Performance:	Tightness Metric = 0.727
	Robustness Metric = 0.002
Runtime:	46.59 s

6.4 Learning structure of the STL formula

Next, we consider the problem of learning STL formulas for the case-studies without using templates. As baseline for comparing the effectiveness and scalability of our incremental approach, we use the genetic algorithm based method described in [8,39]. The timeout was set to 3600 s (1 h).

Case-study	STL property	Runtime (s) of proposed approach	Runtime (s) of Genetic algorithm
$x(t) = t\sin(t^2)$	k=0	929.5	825.7
	k=1	1847.2	1479.2
	k=2	1928.5	1723.6
	k=3	2162.4	2489.2
	k=4	2189.2	3005.7
	k=5	2382.7	3487.5
	k=6	2412.6	TO
	k=7	2591.2	TO
	k=8	2719.4	TO
	k=9	2847.2	TO
	k=10	TO	TO
	k=11	TO	TO
Two agent surveillance	Intruder detect	724.5	398.2
	Non-collision	592.3	487.5
Autonomous car	Property 1	2283.9	TO
	Property 2	1937.6	TO
	Property 3	728.3	2048.1
	Property 4	794.8	1639.5
	Property 5	TO	TO

7 Conclusion

In this paper, we presented a novel approach to learn tight STL formula using only positive examples. Our approach is based on a new tightness metric that uses smooth functions. The problem of learning tight STL properties admits a number of pareto-optimal solutions. We would like to add the capability of specifying preference in which parameters are tightened. Further, computation of the metrics on traces over optimization can be easily parallelized. Another dimension is to study other metrics proposed in literature to quantify conformance and extend tightness over these metrics [10,21]. In conclusion, `TeLEx` automates the learning of high-level STL properties from observed time-traces.

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